

UCT-TP 230/96  
February 9, 2008

# Hadronic Ratios and the Number of Projectile Participants.

Jean Cleymans and Azwinndini Muronga

Department of Physics, University of Cape Town, Rondebosch 7700,  
South Africa

## Abstract

We investigate the dependence of hadronic ratios on the number of projectile participants using a thermal model incorporating exact baryon number and strangeness conservation. A comparison is made with results from  $Au - Au$  collisions obtained at the BNL-AGS.

Preliminary results on the dependence of hadronic ratios on the number of projectile participants have recently been presented by the E866 collaboration [1] for relativistic  $Au - Au$  collisions at the BNL-AGS. These results give insight into the behaviour of the produced hadronic system as a function of the baryon number and of the size of the interaction volume.

It is the purpose of the present paper to analyze these results using a thermal resonance gas model at a fixed temperature and a fixed baryon density. Our treatment differs from previous [2, 3] ones in that we consider the baryon content exactly. This means that we do not introduce chemical potentials for the baryon number (nor for strangeness). Chemical potentials are usually introduced to enforce the right quantum numbers of the system in an average sense. This is a correct treatment for a large system, however, for a small system the production of e.g. an extra proton - anti-proton pair will clearly be more suppressed than in a large system. These extra corrections were first pointed out by Hagedorn [4] and subsequently a complete treatment was presented by many people [5, 6, 7, 8]. We emphasize that these corrections do not contain any information about the dynamics. They simply follow from baryon number conservation. These corrections must be

taken into account before considering more involved models. It is also worth emphasizing that they do not involve any new parameters.

As an example we analyze the (preliminary) data recently presented by the E866 collaboration [1] at BNL.

The exact treatment of quantum numbers in statistical mechanics is obtained by projecting the partition function onto the desired values of  $B$  and  $S$

$$Z_{B,S} = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iB\phi} \frac{1}{2\pi} \int_0^{2\pi} d\psi e^{-iS\psi} Z(T, \lambda_B, \lambda_S) \quad (1)$$

where the usual fugacity factors  $\lambda_B$  and  $\lambda_S$  have been replaced by :

$$\lambda_B = e^{i\phi} \quad \lambda_S = e^{i\psi}. \quad (2)$$

We will use

$$B = 2N_{pp} \quad (3)$$

where  $N_{pp}$  is the number of projectile participants with the factor 2 reflecting the symmetry of the  $Au - Au$  collision system. As the contributions always come pairwise for particle and anti-particle the fugacity factors will give rise to the cosine of the angle. In the further treatment it is useful to group all particles appearing in the Particle Data Booklet [10] into four categories depending on their quantum numbers (we leave out charm and bottom).  $Z_K$  is the sum (given below) of all mesons having strangeness  $\pm 1$  ( $K, \bar{K}, K^*, \dots$ ), similarly  $Z_N$  is the sum of all baryons and anti-baryons having zero strangeness,  $Z_Y$  is the sum of all hyperons and anti-hyperons while  $Z_0$  is the sum of all non-strange mesons, and so on :

$$\begin{aligned} Z_K &= \sum_{j \in |S|=1, |B|=0} V \int \frac{d^3p}{(2\pi)^3} e^{-E_j/T}, \\ Z_N &= \sum_{j \in |S|=0, |B|=1} V \int \frac{d^3p}{(2\pi)^3} e^{-E_j/T}, \\ Z_Y &= \sum_{j \in |S|=1, |B|=1} V \int \frac{d^3p}{(2\pi)^3} e^{-E_j/T}, \\ Z_0 &= \sum_{j \in |S|=0, |B|=0} V \int \frac{d^3p}{(2\pi)^3} e^{-E_j/T}. \end{aligned} \quad (4)$$

We do not include cascade particles as their contribution is unimportant for the energy range under consideration and their inclusion considerably complicates the formalism. Each term will be multiplied by the cosine of an angle, either  $\phi$  or  $\psi$ , in the case where two angles are needed (e.g. for the hyperons) one introduces a new one,  $\alpha$ , using

$$\delta(\phi - \psi - \alpha) = \sum_{n=-\infty}^{\infty} e^{in(\phi - \psi - \alpha)}. \quad (5)$$

Using the integral representation of the modified Bessel functions

$$I_n(z) = \frac{1}{\pi} \int_0^\pi e^{z \cos \theta} \cos n\theta \, d\theta \quad (6)$$

one can write the partition function as

$$Z_{B,S} = Z_0 \sum_{n=-\infty}^{\infty} I_n(2Z_Y) I_{n+B}(2Z_N) I_n(2Z_K) \quad (7)$$

In order to discuss the particle abundances it is useful to introduce the following quantities [8]

$$\begin{aligned} R_K &= \sum_{n=-\infty}^{\infty} I_n(2Z_Y) I_{n+B}(2Z_N) I_{n+1}(2Z_K), \\ R_N &= \sum_{n=-\infty}^{\infty} I_{n+1}(2Z_Y) I_{n+B-1}(2Z_N) I_n(2Z_K), \\ R_Y &= \sum_{n=-\infty}^{\infty} I_{n+1}(2Z_Y) I_{n+B}(2Z_N) I_n(2Z_K), \\ R_{\bar{K}} &= \sum_{n=-\infty}^{\infty} I_n(2Z_Y) I_{n+B}(2Z_N) I_{n-1}(2Z_K), \\ R_{\bar{N}} &= \sum_{n=-\infty}^{\infty} I_{n+1}(2Z_Y) I_{n+B+1}(2Z_N) I_n(2Z_K), \\ R_{\bar{Y}} &= \sum_{n=-\infty}^{\infty} I_{n-1}(2Z_Y) I_{n+B}(2Z_N) I_n(2Z_K). \end{aligned} \quad (8)$$

If a particle,  $i$ , has strangeness 1 and baryon number 0, its density will be given by

$$n_i = \left[ Z_0 \frac{R_K}{Z_{B,S}} \right] \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}. \quad (9)$$

while a particle with strangeness 0 and baryon number 1, will have a density given by

$$n_i = \left[ Z_0 \frac{R_N}{Z_{B,S}} \right] \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T} \quad (10)$$

All other particle densities are obtained by using the appropriate  $R$  factor given in equation (8). The factor in square brackets replaces the fugacity in the usual grand canonical ensemble treatment [2, 3]. Having thus determined all particle densities, we consider the behaviour at freeze-out time. In this case all the resonances in the gas are allowed to decay into lighter stable particles. This means that each particle density is multiplied with its appropriate branching ratio (indicated by  $Br$  below). The abundances of particles in the final state are thus determined by :

$$n_{\pi^+} = \sum n_i Br(i \rightarrow \pi^+),$$

$$\begin{aligned}
n_{K^+} &= \sum n_i Br(i \rightarrow K^+), \\
n_{\pi^-} &= \sum n_i Br(i \rightarrow \pi^-), \\
n_{K^-} &= \sum n_i Br(i \rightarrow K^-), \\
n_p &= \sum n_i Br(i \rightarrow p), \\
n_{\bar{p}} &= \sum n_i Br(i \rightarrow \bar{p}).
\end{aligned} \tag{11}$$

where each sum runs over all particles contained in the hadronic gas.

The comparison with experimental results is shown in figures 1 to 4. To compare with earlier calculations [2, 3] we keep the temperature  $T$  and the baryon density  $B/V$  fixed. This corresponds to keeping the baryon chemical potential fixed in the standard hadronic gas calculations using the grand canonical ensemble.

In figure 1 we compare our results with recent data from the AGS [1, 11]. Figure 1 shows the  $K^+/\pi^+$  ratio. As one can see the results obtained from our calculation show a steep rise with  $N_{pp}$  before leveling off. The dependence on the baryon density is minimal in this case. This result is confirmed by calculations done in the grand canonical ensemble which also show that this ratio is almost independent of the baryon density [3]. We note that the experimental data indicate a slower rise than the model calculation.

In figures 2, 3 and 4 we show the  $K^-/\pi^+$ , the  $K^+/K^-$  and the  $p/\pi^+$  ratios. In each case good agreement is obtained with the results of the E866 collaboration [1]. The relevant temperature is around  $T \approx 100\text{MeV}$ , the baryon density is in the range  $B/V \approx 0.02 - 0.05 \text{ fm}^{-3}$ . In the grand canonical ensemble this corresponds to a baryon chemical potential of  $\mu_B \approx 540 \text{ MeV}$ .

**Acknowledgments** We gratefully acknowledge the financial support of the University of Cape Town (URC) and of the Foundation for Research development (Pretoria). We acknowledge the help of Duncan Elliott for his analysis of the data in the grand canonical ensemble.

## References

- [1] E866 Collaboration, Y. Akiba, Talk presented at Quark Matter '96, Heidelberg May 1996.
- [2] P. Braun-Munzinger, J. Stachel, J. P. Wessels and N. Xu, *Phys. Lett.* B344 (1995) 43.
- [3] J. Cleymans, D. Elliott, H. Satz and R.L. Thews CERN preprint CERN-TH/95-298, *Z. Phys.* C(to be published).
- [4] R. Hagedorn, CERN yellow report 71-12 (1971).

- [5] R. Hagedorn, K. Redlich, *Z. Phys.* C27 (1985) 541.
- [6] C. Derreth, W. Greiner, H.-Th. Elze and J. Rafelski, *Phys. Rev.* C31 (1985) 360.
- [7] M. Gorenstein, V.K. Petrov and G.M. Zinovjev, *Phys. Lett.* B106 (1981) 327.
- [8] J. Cleymans, E. Suhonen, G.M. Weber, *Z. Phys.* C53 (1992)485.
- [9] J. Cleymans and H. Satz, *Z. Phys.* C57 (1993) 135.
- [10] Review of Particle Properties, *Phys. Rev.* D50 (1994) 1177.
- [11] G.S.F. Stephans, “*Strangeness in Hadronic Matter*”, ed. J. Rafelski, AIP Press 1995.

**Figure Captions.**

Figure 1 : The  $K^+/\pi^+$  ratio as a function of the number of projectile participants  $N_{pp}$ . The solid line is obtained for  $T = 96$  MeV and  $B/V = 0.024$  fm<sup>-3</sup>, the dashed line corresponds to  $T = 103$  MeV and  $B/V = 0.050$  fm<sup>-3</sup> while the dotted line corresponds to  $T = 100$  MeV and  $B/V = 0.04$  fm<sup>-3</sup>.

Figure 2 : The  $K^-/\pi^+$  ratio as a function of the number of projectile participants  $N_{pp}$ . The notation is the same as in figure 1.

Figure 3 : The  $K^+/K^-$  ratio as a function of the number of projectile participants  $N_{pp}$ . The notation is the same as in figure 1.

Figure 4 : The  $p/\pi^+$  ratio as a function of the number of projectile participants  $N_{pp}$ . The notation is the same as in figure 1.

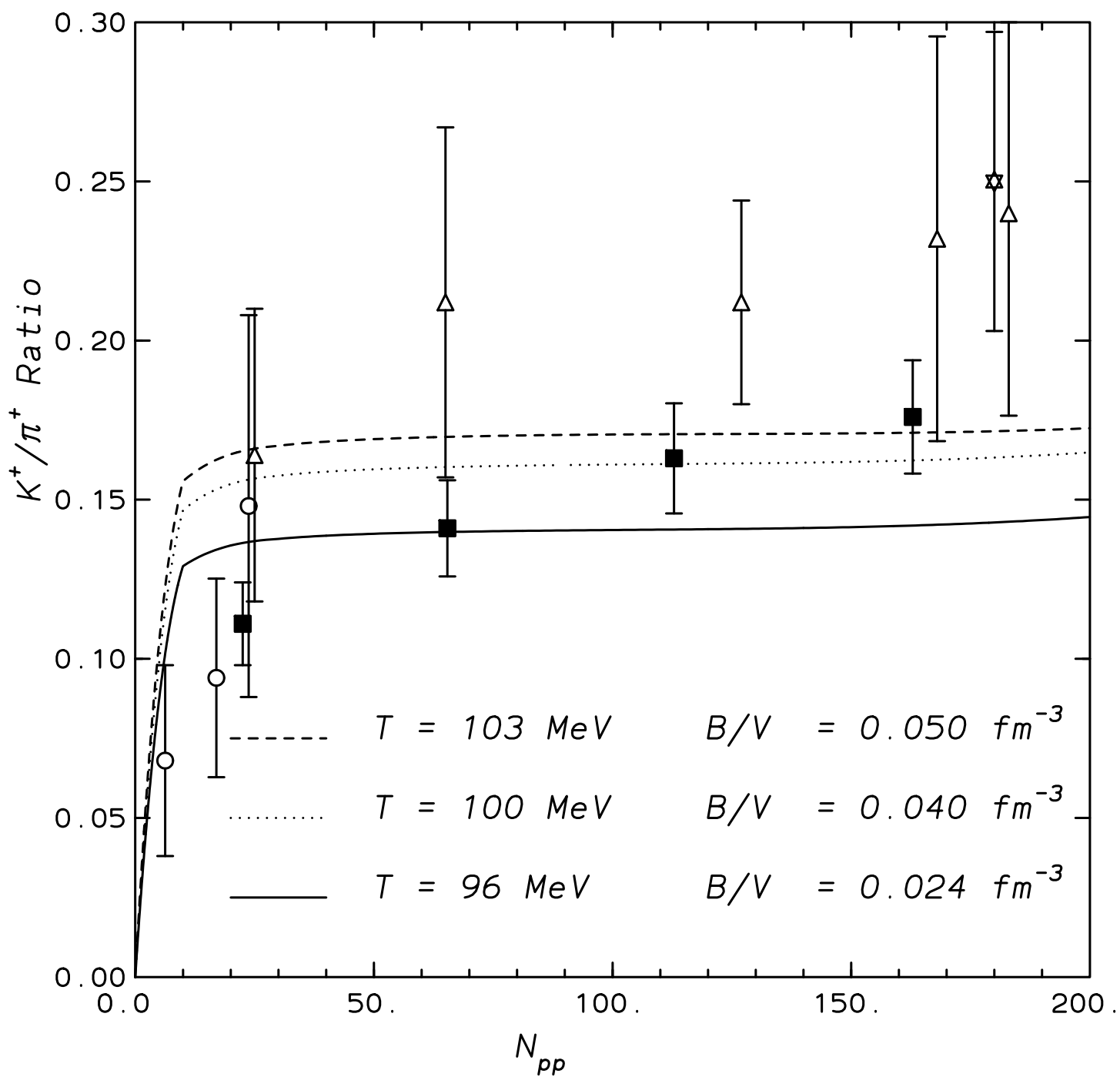


Figure 1

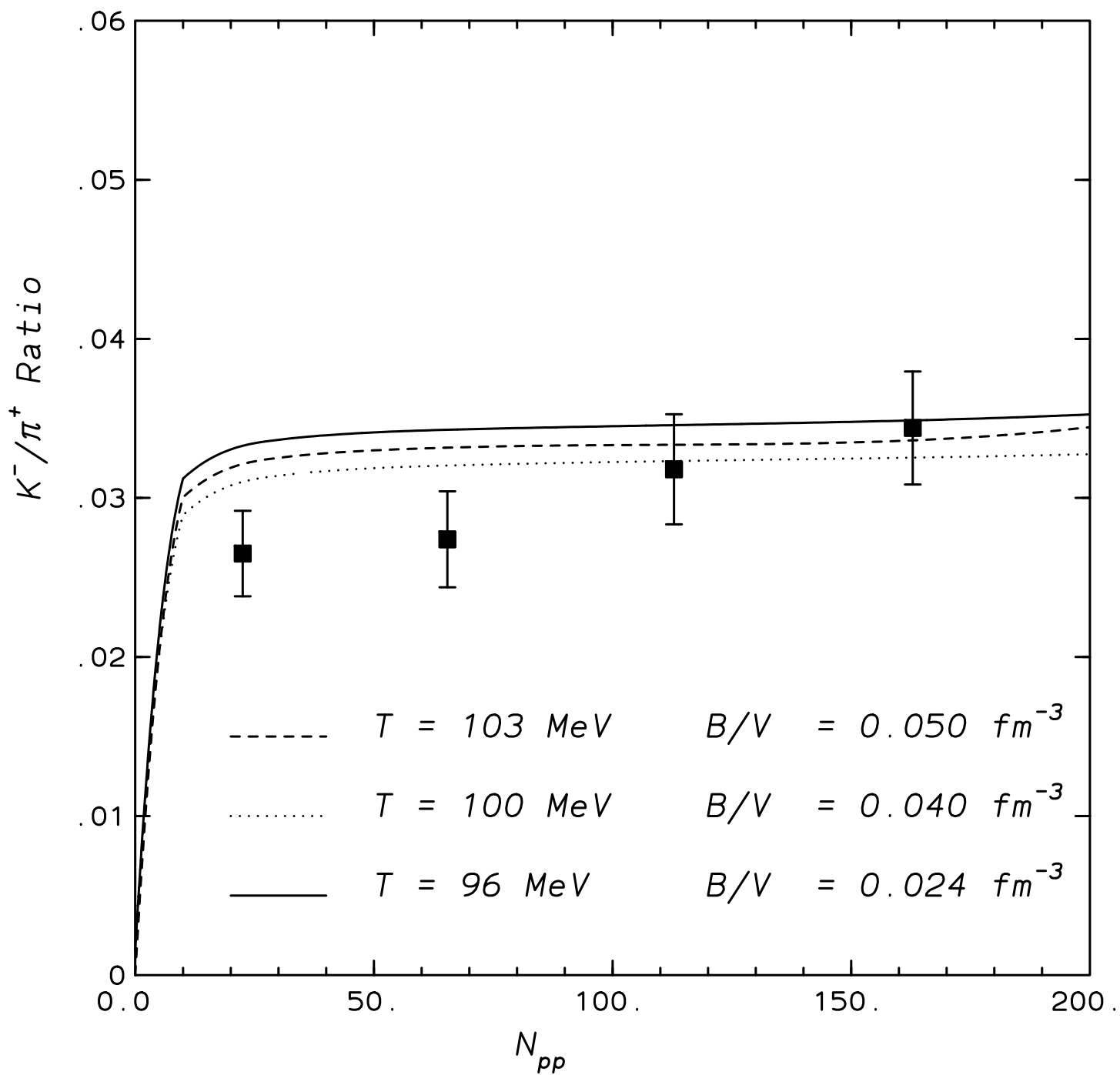


Figure 2



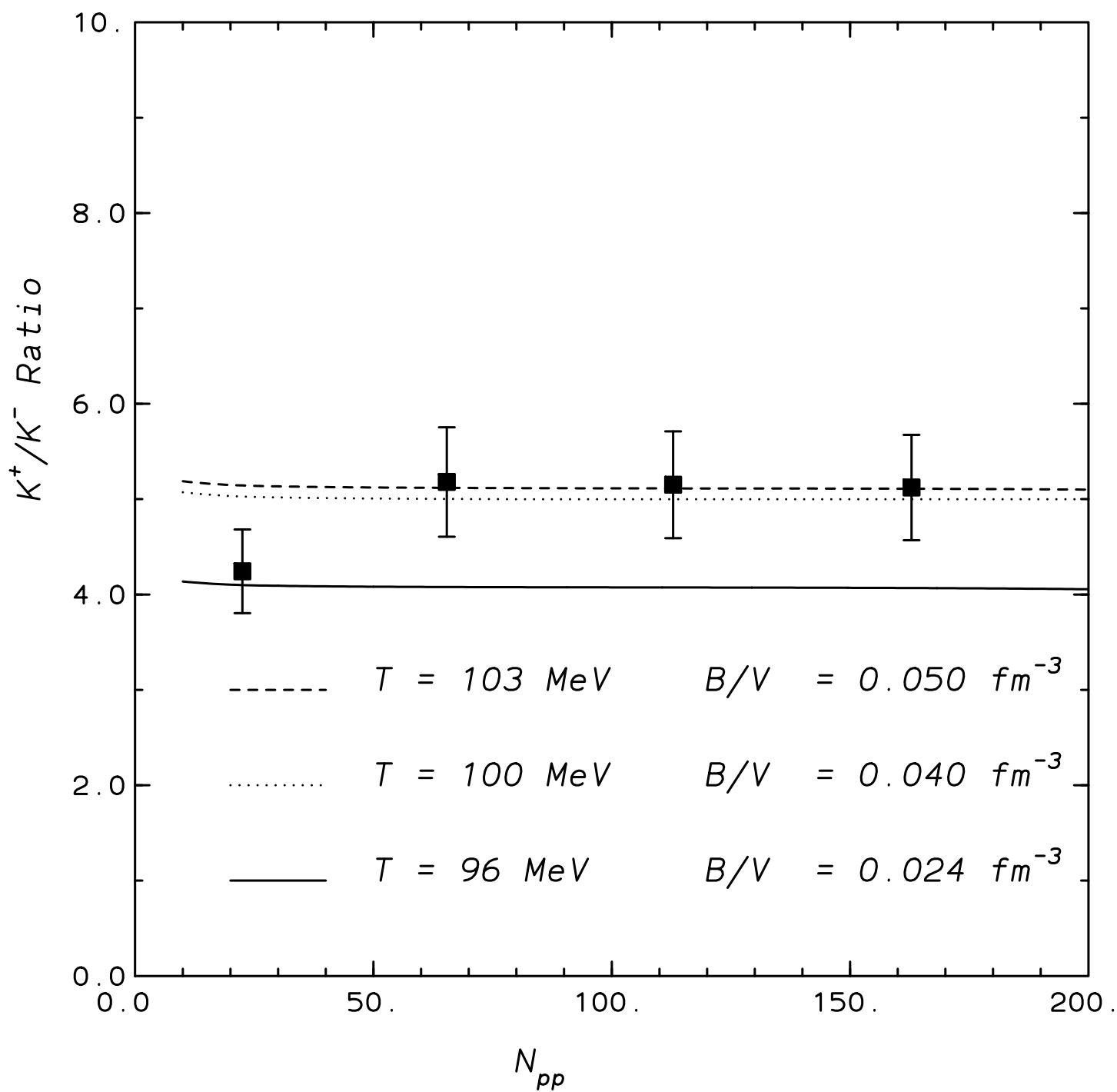


Figure 3

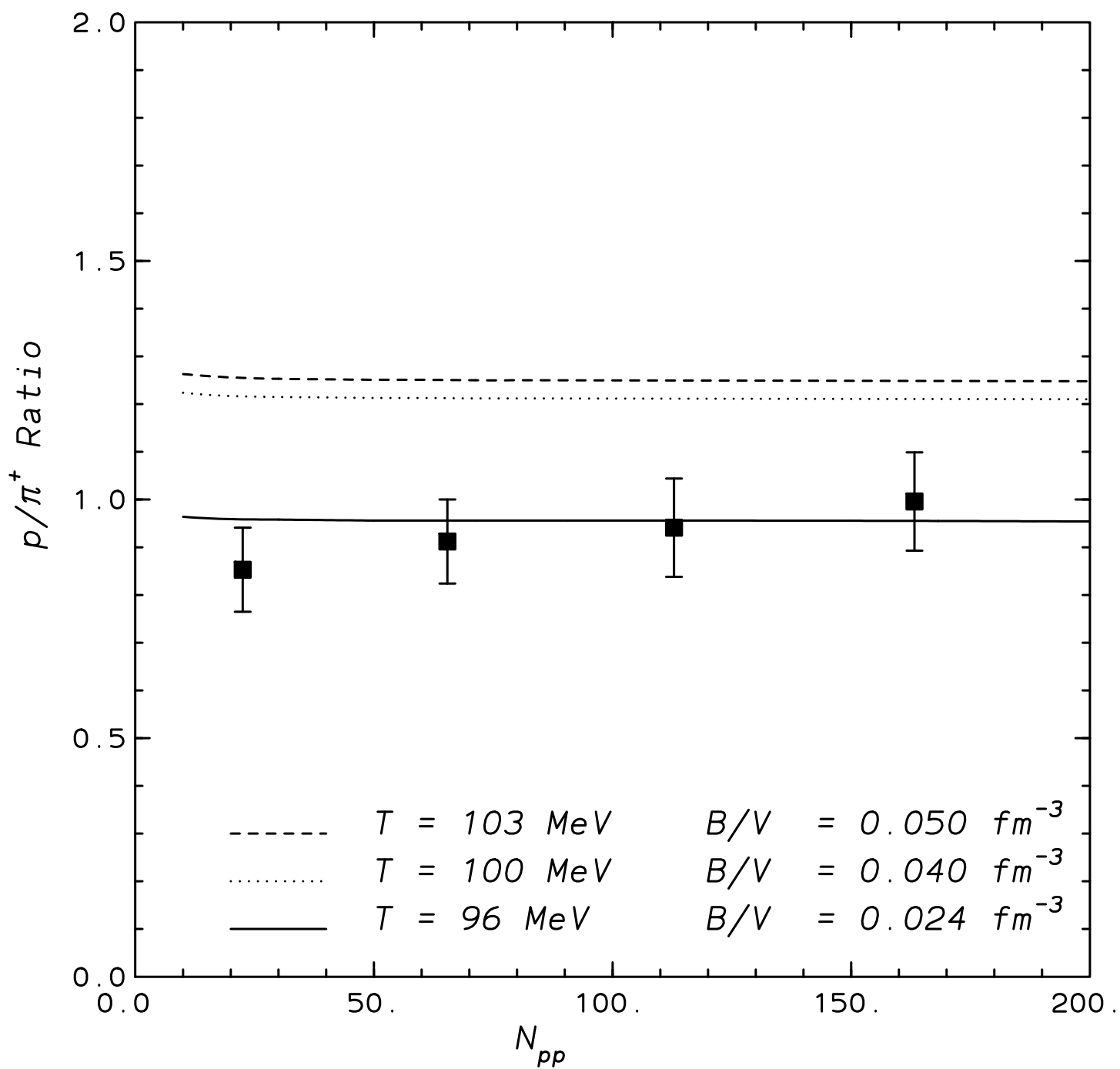


Figure 4